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ABSTRACT

The paper shows that the study of mathematics can be organized as a complex learning enterprise integrating manipulative-exploratory play into a newer software tool environment--a dynamic geometry, a spreadsheet, and a rel^tion grapher. The discussion reflects work done in a lab setting with preservice and inservice teachers enrolled in contemporary general mathematics and problem-solving courses. The psychological aspects of learning mathematical concepts through integrating computer activities and some possible implications of the approach for mathematics teacher education are highlighted from a Vygotskian perspective. (Author/MKR)

Bridging Manipulative-Exploratory Play and the Development of Mathematical Concepts in a Technology-Rich Environment

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BRIDGING MANIPULATIVE-EXPLORATORY PLAY AND THE DEVELOPMENT OF MATHEMATICAL CONCEPTS IN A TECHNOLOGY-RICH ENVIRONMENT

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The paper shows that the study of mathematics can be organized as a complex learning enterprise integrating manipulative-exploratory play into a newer software tool environment — a dynamic geometry, a spreadsheet, and a relation grapher, and it reflects work done in a lab setting with preservice and inservice teachers enrolled in contemporary general mathematics and problem solving courses. The psychological aspects of learning mathematical concepts through integrating off and on computer activities and possible implications of the approach for mathematics teacher education are highlighted from Vygotskian perspective.

The role of play in learning abstract structures has received much attention in educational psychology research. Particularly, Dienes (1964) studied children's learning of mathematical concepts from experiences with concrete materials under the assumption that play and the higher cognitive activities are closely connected. With the advent of advanced technology, it has become considered helpful to use suitably designed computer-based simulations of concrete materials in the learning of mathematics (Thompson, 1992; Kaput, 1994; Steffe & Wiegel, 1994). These uses of a computer, however, involve topics not beyond the elementary and middle levels. The appearance of newer software tools with their tremendous potential for promoting the spirit of exploration and discovery in mathematics classrooms makes it possible to extend the use of concrete materials to more advanced levels of mathematics and to consider play associated with both off and on computer activities. Note we consider the notion of play in the spirit of Hoyles and Noss (1992); that is, student engagement into a play within a learning environment implies exploration, experimentation, wondering about, and enjoyment.

The paper suggests that integrating manipulative-exploratory play into a multiple-application environment enhances the study of advanced mathematical concepts and highlights three essential functions of a computer as a learning medium. First, the variety of available colors and shapes places the choice of manipulatives under the control of learners, and this may strengthen their constructive activity and smooth possible differences in the perception and conceptualization of color and shape (Ratner 1991). Second, the computer environment takes into account indistinct boundaries of manipulative play which may quite imperceptibly move over to an exploration (Dienes, 1964). Manipulative-exploratory play is, in fact, a search for regularities, something that may become an object of manipulation at a higher level. This suggests that the third function of a computer in this setting is to provide the learner with an opportunity of instant transfer from screen images to computing activities and back; that is, the use of appropriate applications integrated into the medium allows the generating of numerical and/or diagrammatic evidence as abstractions from a number of simultaneously scrutinized concrete situations presented by these images. Regularities can then be studied again through play at a higher level of cognitive activity. In addition to these functions, the approach considers learning to be deeply anchored in interactive instruction and emphasizes the role of a teacher in developing students' mathematics knowledge. This role assumes a teacher to be a partner in advancement, one who links small-group explorations and whole class discussion, and mediates the spirit of mathematics learning through a mutually enriching teacher-student dialogue. An equal partnership in such dialogue contributes to the learning of being a reciprocal activity (Confrey, 1995), something that affects both student curiosity and teacher intelligence.

Environment for approaching Fibonacci numbers

The realistic mathematics education argues for instruction to be a process that emphasizes the importance "to recognize a mathematical concept in, or to extract it from, a given concrete situation" (Ahlfors et al., 1962, p.190). A relevant context for accommodating such instructional philosophy is Fibonacci numbers. To approach the concept we suggest to students the following play activities: coloring buildings of different stories, making offspring in the rabbit problem, cutting a square and rearranging the parts in the so-called paradox problem. More specifically, students are engaged in the following phenomenological explorations.

- Exploration 1. Buildings of different numbers of stories are given and one may color them with a fixed color in such a way that no consecutive stories are colored with it. How many different ways of coloring one, two, three, four, etc.-storied buildings are possible?
- Exploration 2. A pair of rabbits is placed in a walled enclosure. Find out how many offspring this pair will produce in the course of a year if each pair of rabbits gives birth to a new pair each month starting from the second month of its life.
- Exploration 3. When you cut a figure and reorder the parts, the shape may change but, the area can not. Consider Figure 1: the square is cut into two congruent triangles and two congruent trapezoids. Can we chose x and y so that the square can be transformed into rectangle as shown?

Within each activity, the same sequence of numbers, known as the Fibonacci sequence, occurs as a result of students' extracting appropriate concepts from manipulative-exploratory play. Once Fibonacci numbers have come into view, they can be explored through spreadsheet modeling; that is, numerical evidence can be used for discovering a number of situations of similar type and extracting an abstraction from these. Moreover, numerical evidence provides a gateway for developing induction proof of the abstraction through visualization with its subsequent symbolization as an important point in the process of learning mathematical concepts (Abramovich, 1995). Thus, the didactical emphasis of the activities is both on conjecturing and developing formal proof rather than on exploring computer-generated patterns "at the expense of discovering their underlying relationships" (Noss, 1994, p.9). Yet the environment accommodates learners of different *zones of proximal development* allowing for the diversity in the pace of activities, in the consuming of teacher-mediated assistance, and in the depth of exploration. Finally, when the mystery of the paradox problem is resolved, the use of a relation grapher enables students to make sense of the concept of the golden ratio. Note that geometric aspects of the paradox problem can be explored both in a traditional setting (paper grid and scissors) and in that of software setting. Comparing off and on computer activities in resolving the problem leads to the following important observation: when manipulating parts of a square within off-computer activities a student uses geometric transformations such as rotation, reflection, and translation almost automatically or spontaneously, yet these capacities lack conscious awareness. Though the student does act consciously in performing transformation, his or her attention is not directed toward the possessing of geometric skill, and its nonvolitional nature is shaped by the structure of the particular situation. On the other hand, the use of a computer allows for the learning of conscious awareness of the same operations while operating software. Therefore one can use this example in order to discriminate an instructional use of manipulatives associated with on and off-computer activities. This distinction is constructed on the lines of Vygotsky (1987) who, using language acquisition as a paradigm case, argued that the role of school instruction in written speech and grammar is to make a child learn "conscious awareness of what he does ... [that is, the child] learns to operate on the foundation of his capacities in a volitional manner" (p. 206). In much the same way, learning to operate dynamic geometry software like *GSP* leads to the mastery of school geometry and plays an important role in this process.

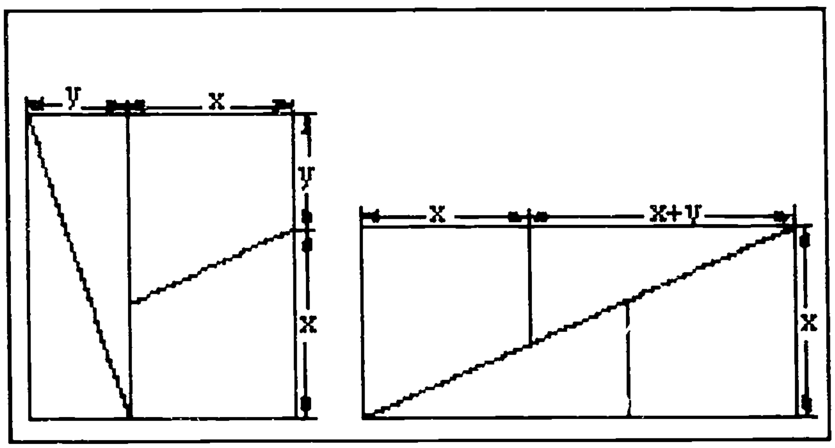

Figure 1. The paradox problem.

♦	♣	♠
♥	♦	♣
♥	♦	♠
♥	♣	♠
C(4,3)=C(3,3)+C(3,2)		

Figure 2. Recursive strategy.

Environment for the study of combinatorics

The study of combinatorics can be nicely orchestrated through integrating manipulative-exploratory play and computing activities. For example, consider the following problem: *In how many ways can three suits be selected from four different suits?* Combinatorial reasoning can be better acquired by clearing away the situation through the use of manipulatives. As one in-service teacher noted: "I feel manipulatives are probably one of the best ideas I have ever seen for showing C(n,r) — combinations of n things taken r at a time. This will be very useful in my upcoming lecture on combinations."

Indeed, in the first stage manipulatives serve as a means for solving the counting task so that abstraction from a number of similar arrangements of manipulatives occur in the form of recursive definition of combinations. In carrying out this task, we first discovered students' involuntary behavior in creating the combinations, something that seems to bring a chaos into the approach as the number of objects increases. Yet, a spontaneous strategy is not a useless experience, but on the contrary, it allows students to reach the threshold in the development of mathematical thinking beyond which conscious awareness of recursive strategy becomes possible. Indeed, when asked to be systematic, students often apply recursive reasoning: when hearts are not in use at all, three remaining suits can be selected in C(3,3) ways; when the heart is in use, two other counterparts can be selected in C(3,2) ways (Figure 2). This strategy possibly lacks conscious awareness of recursive thinking, though, in fact, this is the case of recursion. In the words of Vygotsky "consciousness and control appear only at a late stage in the development of a function, after it has been used and practiced unconsciously and spontaneously. In order to subject a function to intellectual control, we must first possess it" (cited in Bruner, 1985, p. 24). The teacher-mediated link between spontaneous and purposeful problem solving strategies thus becomes crucial for 'good learning,' for it is the link of the zone of actual and proximal development of the learners.

The next step in the study of combinations involves setting up on a spreadsheet boundary conditions for combinations obtained through manipulative-exploratory play and modeling them using a recursive nature of software. It is worth noting that the language of communication with the software might be that of pointing to cells (a kind of the substitution of speech for concrete action) rather than solely formula-based, and this allows students to shift the onus of both symbolization and generalization onto a spreadsheet. In such a way, the software serves as a support system that helps learners to make a non-algebraic leap from empirical data linked by an intuitive guess to the numerical projection of its generalization (modeling data). Once a large pool of combinations come into view the activities focus on making connections among combinations and testing these connections in terms of manipulatives. In other words, the activities deal with creating visual proofs of combinatorial propositions. One may note, for example, that $C(4,3)=C(4,1)$, or $C(4,3)=C(3,3)+C(2,2)+C(2,1)$ and then justify these findings using manipulatives (visual proofs). In doing so, one is engaged into a play on a higher level of cognitive activity using, in fact, the same concrete embodiments that allowed for the reaching of this level. Transferring from a special case of identities involving numbers with combinatorial meaning to their general form results in students involvement in the development of inductive proofs of the identities, mathematical activity stimulated and guided by computer-generated numerical evidence.

Note that although, as observation shows, the task to discover Fibonacci numbers among combinations (both through exploring numerical patterns on a spreadsheet and imparting combinatorial meaning to the coloring task) proves to be a challenge for most of the students, the principle of "raising the ante" of the task (Bruner, 1985) allows for maintaining students' interest in developing mathematical concepts and for demonstration of the endless mathematical explorations through the intertwining of different learning strands.

Assessment through reciprocal teaching

The environment described in this paper may have important implications for an assessment practice that incorporates reciprocal teaching (Palincsar & Brown, 1984). We applied this procedure for final sessions by splitting students enrolled in a problem solving class into equal groups, each of which was assigned to create a task for an associate group. The instructional goal was to demonstrate how the environment allows for students' affluent and seemingly endless performance on a regular task and encourages the development of mathematical ideas that are far beyond the task's original design. The sessions have shown that *all* students may become motivated and challenged by learning mathematics, provided that a classroom environment is conducive to students' pursuing avenues of personal interest and attaining ownership of concepts discovered. We conclude the paper with a hope that computer-enhanced reciprocal teaching embodies N.C.T.M.'s (1995) vision of an assessment as "a dynamic process that informs teachers...and supports each student's continuing growth in mathematical power" (p.6).

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